WAVE OPTICS

Ashika A

(1)

Principle of Superposition: when a no. of waves travelling thru' a medium superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of the displacements due to individual waves.

If $\vec{y_1}, \vec{y_2}, \vec{y_3}$ are the displacements due to diff. waves ~~acting~~ passing simultaneously thru' a medium, then resultant displ. at a pt is

$$\vec{y} = \vec{y_1} + \vec{y_2}$$

~~Coherent sources~~ When the 2 superposing waves ~~reach at pt~~ are in same phase, (ie crest of one wave falls on crest of the other → or trough of the other) then their displacements get added.

→ When they meet at a pt. in opp-phase ~~then~~ ie crest of one falls on trough of other or vice versa) then their displacements get subtracted.

Coherent Sources - Two sources of light which emit light waves of same wavelength, frequency, with same or const ph. diff. → Coherent Sources.

Consider the displacements of 2 waves from 2 coherent sources S_1 and S_2 at any instant - then, if ϕ is the ph. diff. b/w the waves.

$$y_1 = a \cos \omega t$$

$$y_2 = a \cos(\omega t + \phi)$$

Resultant displ. $\vec{y} = \vec{y_1} + \vec{y_2}$

$$= a \cos \omega t + a \cos(\omega t + \phi)$$

Interference is the redistribution of intensity of light waves due to superposition of waves. Interference phenomenon of redistribution of light energy in the surrounding space as a result of superposition of light waves from coherent sources.

(2)

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$= 2a\cos(\phi/2)\cos(\omega t + \phi/2)$$

As amplitude of the resultant displ. is $2a\cos\phi/2$, $\therefore$ result. intensity will be

$$I = 4a^2\cos^2\phi/2$$

$$= 4I_0\cos^2\phi/2.$$

If $\phi = 0, \pm 2\pi, \pm 4\pi \ldots$ corresponds to the condition of constructive interference, leading to maximum intensity

i.e. $I = 4I_0$.

If $\phi = \pm\pi, \pm 3\pi, \pm 5\pi \ldots$ corresponds to condition of destructive interference leading to zero intensity. Thus we can see a stable interference pattern at a screen placed at a distance from the source. ~~In terms of p.d.~~ Thus we can see positions of maxima & minima corresponding to const. & destr. interference or a stable int. pattern on a screen placed at a distance from the source. A phase diff. of 2π corresponds to a p.d. of λ.

$\Delta = n\lambda$ (integral multiple of λ).

i.e. phase diff, $\phi = \frac{2\pi}{\lambda} \times$ p.d. $\quad \phi = 2n\pi$ (in. m of 2π)

$\therefore$ The condition for constructive interference is when light from 2 coherent sources reaching a point has a p.d. equal to $n\lambda$

and the condition for destructive interference will be p.d. equal to $\Delta = (n+\frac{1}{2})\lambda$.

$\phi = (2n+1)\pi$ → odd multiple of π. ($\frac{1}{2}$ integral multiple of λ)

* If the two sources are not coherent i.e. they do not have a const ph. diff. b/w them, the interference pattern will not be clear i.e. positions of maxima & minima will change

and hence a time averaged intensity can be seen which is given by

$$\langle I \rangle = 4I_0 \langle \cos^2 \phi/2 \rangle$$

$$\langle \cos^2 \phi/2 \rangle = 1/2$$

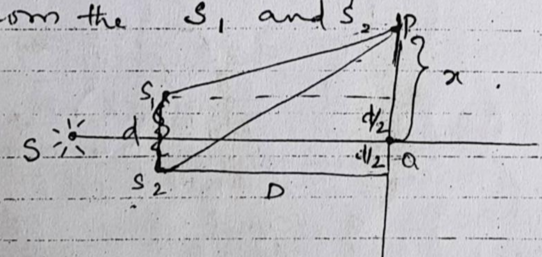
$\therefore \langle I \rangle = 2I_0 \longrightarrow$ will be the resultant intensity at all points. (that is why we don't see any interference pattern when 2 light sources illuminate a wall).

Conditions for sustained interference

- ⊛ The two light sources should be coherent i.e it should emit light of same ν, ~~same λ~~ and have const. phase difference.
- ⊛ Monochromatic - or same λ → diff. colours produce diff. interference pattern. Eg: sodium vapour lamp
- ⊛ Two sources should be narrow
- ⊛ Distance b/w source & screen should be large.

Interference of light waves & Young's double slit Expt.

Consider two point sources S_1 and S_2 separated by a small distance 'd' be illuminated by a source S. Let a screen be placed at a distance D from the S_1 and S_2.



③

S_1 and S_2 will act as two coherent sources producing a stable int. pattern on the screen. Thus any arbitary point P at a distance x from O (centre of the screen) will be position of maximum depending on the condition that path difference.

$$S_2P - S_1P = n\lambda \quad \text{where } n = 0, 1, 2, 3 \ldots$$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 - \left(x - \frac{d}{2}\right)^2\right]$$

$$= 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

as $d << D$; we can take

$$S_2P \sim S_1P \quad \text{or} \quad S_2P + S_1P = 2D$$

$$\frac{xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{d}$$

$$\therefore \text{path diff} = \frac{\cancel{2}xd}{\cancel{2}D} = n\lambda.$$

$\therefore$ Hence point P will be of max. intensity bright region when

$$\boxed{x = x_n = \frac{n\lambda D}{d}} \; ; \; n = 0, \pm 1, \pm 2, \ldots$$

& ~~etc~~ for dark region, we can write

$$\boxed{x = x_n = \left(n + \frac{1}{2}\right)\frac{\lambda D}{d}}$$

where $n = 0, \pm 1, \pm 2 \ldots$

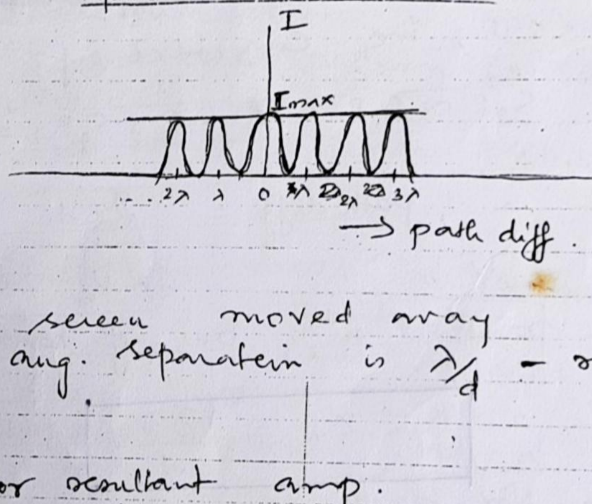
Thus alternate bright and dark bands of eq width appear on ~~either~~ the screen called fringes which are equally spaced on either side of central point O. Width of the fringe — is the distance b/w 2 consecutive bright or dark fringes

$$\beta = x_{n+1} - x_n$$

$$\boxed{\beta = \lambda D/d} \longrightarrow \text{fringe width.}$$

* Central point is always bright - as waves from S_1 and S_2 will always arrive in phase at O as S_1O and S_2O are equidistant

Intensity distribution Curve for Interference



* when screen moved away
$\rightarrow$ ang. separation is λ/d - remains const.

Exp. for resultant amp.

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

* No two independent source can act as coherent (T) sources because emission of light is due to millions of atoms. So phase diff. b/w waves emitted by diff. atoms will change randomly & sustained interference will not be obtained.

* width of central bright fringe is $\beta_0 = \frac{D\lambda}{d}$.

* If YDS apparatus is immersed in a liquid of ref. index μ, then wavelength λ' reduces to λ/μ.

$$\therefore \quad \beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{\mu d} = \beta/\mu$$ ie fringe width decreases.

* (YDS expt. can be used to measure wavelength of eg a monochromatic light)

* sustained interference $\longrightarrow$ positions of maxima & minima do not change with time or it produces stable interference pattern.

* $$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \left(\frac{r+1}{r-1}\right)^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \frac{w_1}{w_2}$

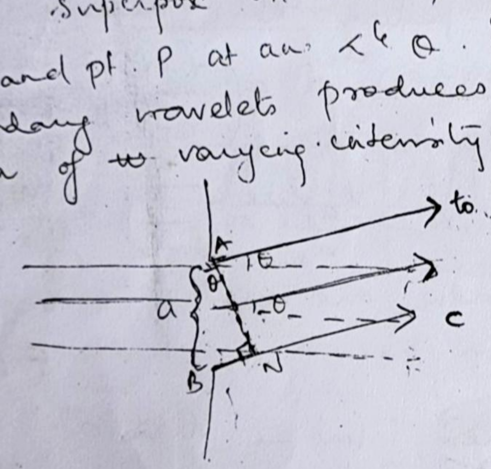
where w_1 and w_2 are width of the slits S_1 and S_2 resp.

Interference by thin films — when oil spreads over a water surface — ~~Colourful~~ we can see display of colours on it - due to interference of light which is reflected from upper & lower surfaces of film.

Diffraction : The phenomenon of bending of light around the corners of small obstacles or aperture and its consequent spreading into the regions of geometrical shadow - diffraction of light.

Eg: when a monochromatic source of light is viewed thru' a piece of cloth, the source appears as an enlarged disc.
light passing thru' a pinhole, is seen as alternate bright & dark bands on a screen placed behind the pinhole.

When a plane wavefront of monochromatic light illuminates the slit AB, each pt. on the slit AB becomes source of secondary wavelets. The secondary wavelets originating from diff points superpose on each other, while travelling towards C, and pt. P at an ∠ θ. Superposition of the secondary wavelets produces a diffraction pattern of varying intensity.



The intensity of central band is maximum & goes on decreasing on both sides.

Let AB be a slit of width 'a' & a parallel beam of monochromatic light is incident on it. Let θ be the ∠ of diffraction for waves reaching point P of screen & AN ⊥r to wavefront diffracted from B.

Then path diff. b/w rays diffracted at pts A and B

$\Delta = BP - AP = BN$

$\sin\theta = \frac{BN}{AN}$ or $BN = AN\sin\theta = a\sin\theta$.

or $\Delta = a\sin\theta$. —— (1)

At the central pt. C of the screen, the ∠ θ is zero. Hence waves arriving from all pts of the wave-front arrive in same phase. This gives max. intensity at C.

If point P on screen is such that path diff. b/w waves starting from ~~at~~ A & B is λ

then $a\sin\theta = \lambda$

or $\sin\theta = \lambda/a$.

or $\theta \sim \lambda/a$.

<u>Minima</u>: If we divide the slit into 2 equal halves AO and OB, each of width $a/2$.
For every pt. M_1 in AO, there is a corresponding pt M_2 in OB, such that $M_1M_2 = a/2$. Then p.d. ~~at~~ b/w waves arriving at P starting from M_1 and M_2 will be

$$\Delta = \frac{a}{2}\sin\theta = \frac{a}{2}\theta = \frac{a}{2}\times\frac{\lambda}{a} = \frac{\lambda}{2}.$$

which means that contributions from the 2 halves of the slit are opp. in phase & hence cancel each other ~~general~~ which gives the angle of diffraction at which intensity falls to zero.

Thus general condition for minima is

$a\sin\theta = n\lambda$.

Secondary Maxima: Consider an angle θ such that $\sin\theta = \theta = \frac{3}{2}\lambda/a$

Secondary Maxima : suppose the point P is so located that $\Delta = 3\lambda/2$.

$$\Delta = a \sin\theta = a\theta = \frac{3}{2}\lambda.$$

We can divide slit into 3 equal parts. The p.d. b/w 2 corresponding points of the 1st 2 parts will be $\lambda/2$, the wavelets from these points which will interfere destructively. The wavelets from the 3rd part of the slit will contribute to some intensity forming a secondary maximum.

∴ Condition for secondary maxima,

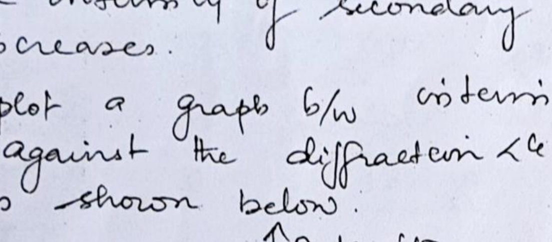
$$a \sin\theta_n = (n + 1/2)\lambda \quad \text{where } n = 1, 2, 3 \ldots$$

directions of secondary max. are

$$\theta_n = (n + 1/2)\lambda/a.$$

The intensity of secondary max. decreases as n increases.

When we plot a graph b/w intensities of max. & minima against the diffraction <le θ, we get a graph as shown below.



→ Intensity of the central maximum is due to constructive interference of wavelets from all parts of the slit,

→ First sec. max. is due to contribution of wavelets from one third part of the slit (wavelets from the remaining part interfere destructively).

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→ Linear width of Central maximum.

$\beta_0 = \frac{2D\lambda}{a}$ (It is the distance b/w the 1st sec. min. on either sides of central point O)

→ Ang. width of Central Maximum, $2\theta = 2\lambda/d$.

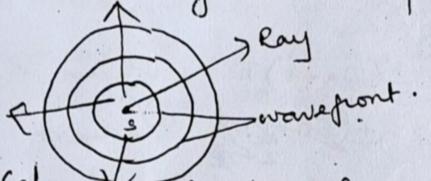
→ width of a sec. max $\propto \frac{1}{\text{Slit width}}$.

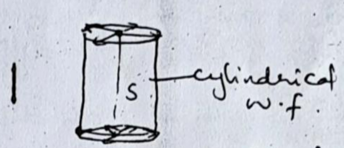
~~a~~ ie a distinct diff. pattern is possible if the slit is very narrow.

Wave Optics Extra points (1) Ashika A
XII

wavefront – Continuous locus of all particles of the medium which are oscillating in phase ie it is a surface of constant phase. The speed with which a wavefront moves is the phase speed. Thus every crest or trough is a wavefront.

spherical wavefront : when waves travel in all directions from a point source, the wavefronts are spherical in shape. As all the points equidistant from the source lie on a sphere – wavefront is spherical.

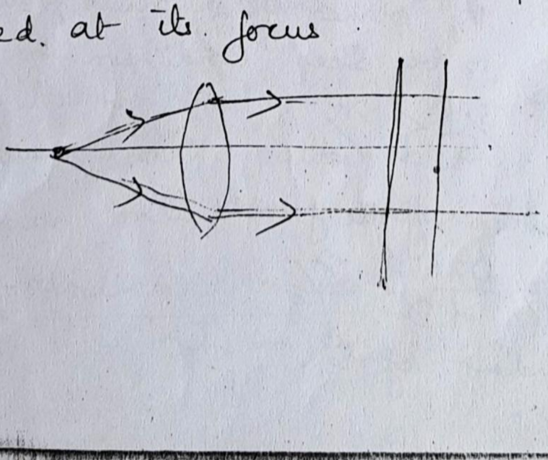




cylindrical wavefront : when the source of light is linear in shape, like a fine rectangular slit, the wavefront is cylindrical in shape because the locus of all such points which are equidistant from the linear source will be a cylinder.

plane wavefront : As a sph. or cyl. w.f. advances, its curvature decreases progressively, ∴ a small portion of the wavefront will appear plane.

Shape of wavefront for (i) Light diverging from point source. (ii) Light emerging ~~from~~ out of a convex lens when a point source is placed at its focus



* Interference (Important points)

Condition for maxima.

ph. diff. $\phi = \pm 2n\pi$ where $n = 0, 1, 2, 3 \ldots$

path diff $\Delta = n\lambda$ where $n = 0, 1, 2, 3 \ldots$

Max. amp; $a_{max} = a_1 + a_2$

Max. Intensity $I_{max} = (a_1 + a_2)^2$

$= a_1^2 + a_2^2 + 2a_1a_2$

$= I_1^2 + I_2^2 + 2\sqrt{I_1 I_2}$

$\Delta\phi \rightarrow$ phase diff

$\Delta \rightarrow$ path diff

Ph. diff $= \frac{2\pi}{\lambda} \times$ p.d

i.e. $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta$

Condition for minima

ph. diff $\phi = \pm(2n+1)\pi$ where $n = 0, 1, 2, 3 \ldots$

path diff $\Delta = \pm \cancel{\frac{(2n+1)}{2}} (n + \tfrac{1}{2})\lambda$ where $n = 0, 1, 2, 3 \ldots$

Min amp; $a_{min} = a_1 - a_2$

Min intensity; $I_{min} = (a_1 - a_2)^2$

$= a_1^2 + a_2^2 - 2a_1a_2$

$= I_1 + I_2 - 2\sqrt{I_1 I_2}$

* When white light is used to illuminate the slit, we obtain an interference pattern with a central white fringe having few coloured fringes on two sides and then uniform illumination.

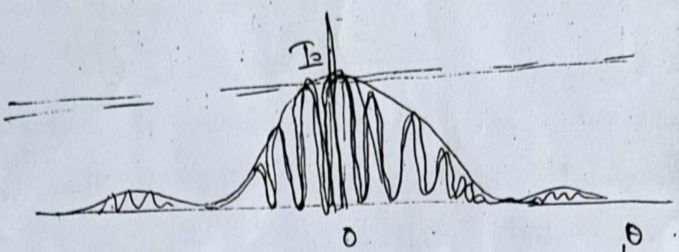
* If s is the size of the source & S the distance of the source from the plane of 2 slits, then for interference pattern to be seen condition to be satisfied is $s/S < \lambda/d$.

* Angular separation of the fringes

$$\theta = \beta/D = \frac{D\lambda}{d\,D} = \lambda/d$$

is independent of D.

→ Interference pattern in a double slit is actually a superpositen of single slit diffraction from each slit.



The diagram shows several fringes due to double slit interference, contained in a broad diffraction peak. when the separation b/w the slits is large compared to their width, diff. pattern becomes very flat and we observe only the 2 slit int. pattern.

* Distinguishing feature b/w diffraction & Interference.

→ For a single slit of width 'a', the first null of diff. pattern occurs at an ∠le of λ/a. At the same ~~[illegible]~~ angle of λ/a, we get a maxima for 2 narrow slits seperated by distance 'a'.

→ The interference pattern has a no. of equally spaced bright and dark bands while diffraction pattern has a central bright maximum which is twice as wide as other maxima.

→ The no. of interference fringes depend on the ratio of the distance b/w the slits to the width of a slit.

→ In the limit of width of a slit becoming v. small, or large, diff. pattern become very flat and observe only int. pattern.

→ The resultant intensity at any pt. on the screen is $\boxed{I = 4I_0 \cos^2 \phi/2}$

→ Diffraction is due to interference of secondary wavelets starting from different parts of the wavefront that passes thru' the aperture.

→ Diffraction pattern cannot be seen with a wide slit illuminated by monochromatic light
– width of central max. is small and variation of intensity by other maxima & minima is also so small that they cannot be distinguished.

→ Diffraction is common in sound waves 'cos the λ of sound waves very large so they get easily diffracted by objects around us while in case of light waves for diffraction to be pronounced, λ should be comparable to size of the obstacle which is not common ~~everyday~~

Wave Optics ①

* The number of interference fringes of smaller width that occur in the broad diffraction peak depends on the ratio d/a → which is the ratio of distance b/w the slits to the slit width.
* If width 'a' becomes exceedingly small ie $a \to 0$, the resulting diffraction pattern becomes too broad. Due to this, we will be able to observe double-slit interference pattern with increased intensities.
* Size of the central maximum is (linear width)

$$2x = \frac{2D\lambda}{a}$$

Angular width of central maximum

$$2\theta_1 = \frac{2\lambda}{a}$$

Here 'x' is the distance of the 1st secondary minimum from the central point 'O' on the screen

or $$x = \frac{\lambda D}{a}.$$

* The location of n^{th} bright fringe on the screen in YDS experiment is (from the centre).

$$x_n = n\beta = \frac{n\lambda D}{d}$$

where $n = 0, 1, 2, 3 \ldots$

λ → wavelength of light used
D → distance b/w source slits screen
d → distance b/w source slits

* The position of n^{th} dark fringe

$$x_n = \left(n - \tfrac{1}{2}\right)\beta = \frac{(2n-1)}{2}\frac{\lambda D}{d}$$

where $n = 1, 2, 3 \ldots$

* The angular separation for n^{th} bright fringe,

$$\theta_n = \frac{n\beta}{D} = n\frac{\lambda}{d}, \quad n = 0, 1, 2, \ldots$$

for n^{th} dark fringe

$$\theta'_n = \left(n - \tfrac{1}{2}\right)\frac{\beta}{D} = \left(n - \tfrac{1}{2}\right)\frac{\lambda}{d}$$

$$n = 1, 2, 3 \ldots$$

* In interference, the energy is redistributed in space, there is no loss or gain in energy.
* The amplitude of waves is $\propto$ to the area of the slit ie $a \propto A$
* when $d = \frac{\beta}{2}$ $\rightarrow$ uniform illumination occurs.
* when one of the slits in YDS is closed, the contrast b/w the fringes decreases.
* If $d < \lambda$, then $\beta > D$ - pattern not practically visible.
* If the coherent sources consists of an object and its reflected image, the central fringe is dark instead of bright.